

Analytical Reconstructions of Regions of Interest in Medical Imaging

Kévin Polisano

Supervised by :
Laurent Desbat

09/18/2012



1 Introduction

- Context
- Topic
- Principles of tomography and motivation

2 Theory

- State of the art
- Incomplete data reconstruction

3 Implementation

- Acquisition
- Rebinning
- Reconstruction

4 Results and analysis

5 Conclusion

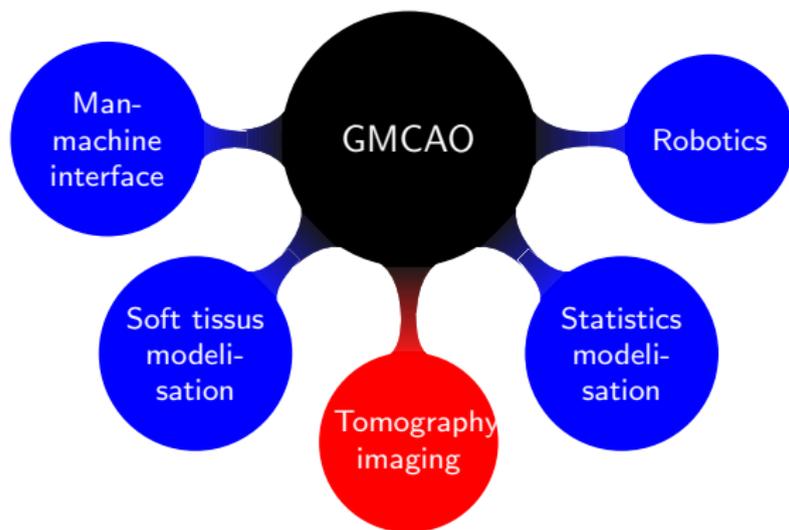
- Personal record
- Future improvements

Contents

- 1 Introduction
 - Context
 - Topic
 - Principles of tomography and motivation
- 2 Theory
- 3 Implementation
- 4 Results and analysis
- 5 Conclusion

Introduction

Context



Clinical applications



Introduction

Topic



What is the topic of this intership ?

- To discover the field of Computed Tomography Imaging
- To improve the current way of reconstruction of images
- To reduce the X-ray exposure by decreasing the trajectory of the scanner around the patient
- To implement a first version in Matlab which could be reused then by Surgivisio

Introduction

Topic



What is the topic of this intership ?

- To discover the field of Computed Tomography Imaging
- To improve the current way of reconstruction of images
- To reduce the X-ray exposure by decreasing the trajectory of the scanner around the patient
- To implement a first version in Matlab which could be reused then by Surgivisio

Introduction

Topic



What is the topic of this intership ?

- To discover the field of Computed Tomography Imaging
- To improve the current way of reconstruction of images
- To reduce the X-ray exposure by decreasing the trajectory of the scanner around the patient
- To implement a first version in Matlab which could be reused then by Surgivisio

Introduction

Topic

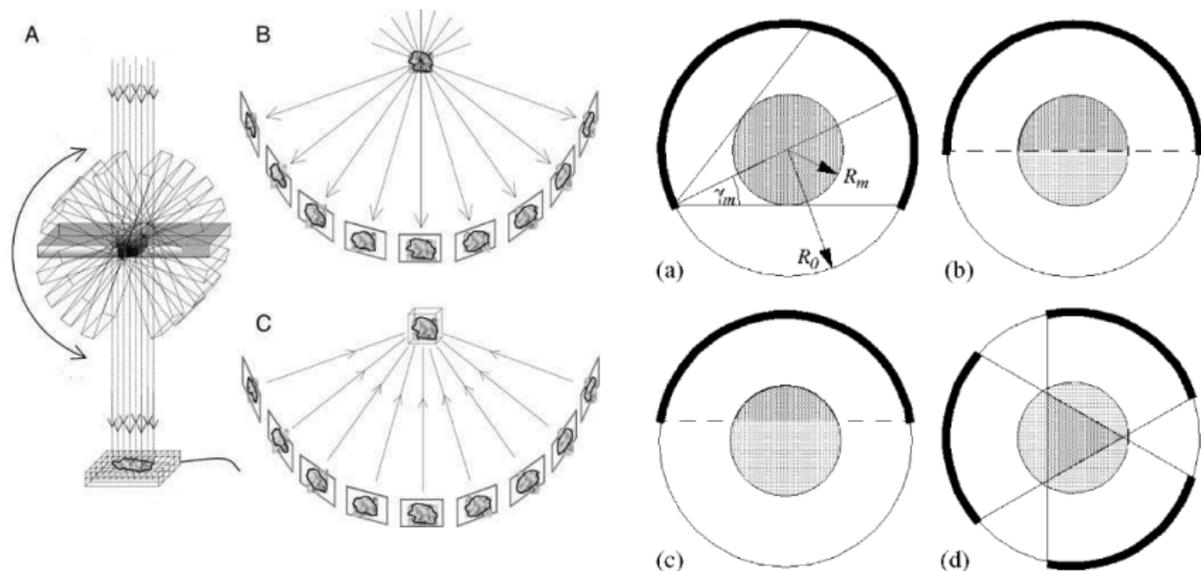


What is the topic of this intership ?

- To discover the field of Computed Tomography Imaging
- To improve the current way of reconstruction of images
- To reduce the X-ray exposure by decreasing the trajectory of the scanner around the patient
- To implement a first version in Matlab which could be reused then by Surgivisio

Introduction

Principles of tomography

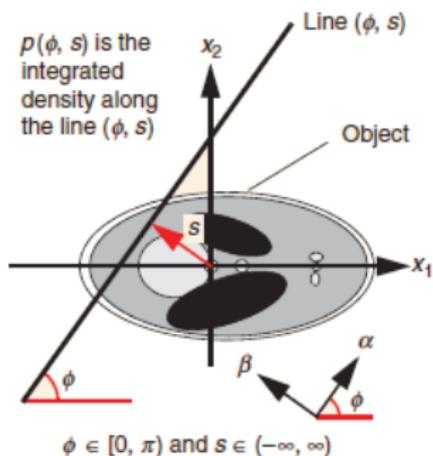


Contents

- 1 Introduction
- 2 Theory
 - State of the art
 - Incomplete data reconstruction
- 3 Implementation
- 4 Results and analysis
- 5 Conclusion

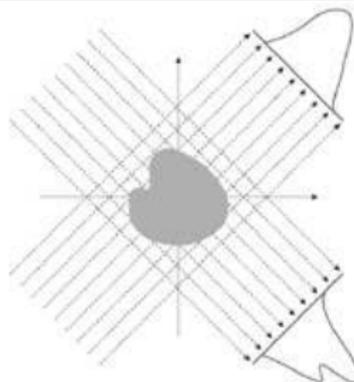
Mathematical formulation

Projection



Projection

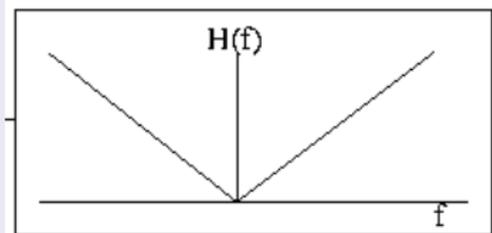
$$p(\phi, s) = \int_{-\infty}^{\infty} f(r\alpha + s\beta) dr \text{ for } \phi \in (0, \pi), s \in (-\infty, \infty)$$



Mathematical formulation

Filters applied to projections

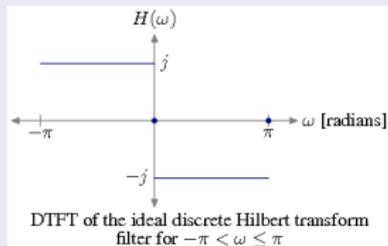
Ramp filter



$$p_R(\phi, \cdot) = p(\phi, \cdot) * r$$

$$R(\sigma) = |\sigma|$$

Hilbert filter



$$p_H(\phi, \cdot) = p(\phi, \cdot) * h$$

$$H(\sigma) = -j \cdot \text{sign}(\sigma)$$

Mathematical formulation

Filtred backprojection

Filtred backprojection

- $f(x) = \int_0^\pi p_R(\phi, s)|_{s=x \cdot \beta} d\phi$
- $f(x) = \frac{1}{2\pi} \int_0^\pi \frac{\partial p_H}{\partial s}(\phi, s) d\phi$

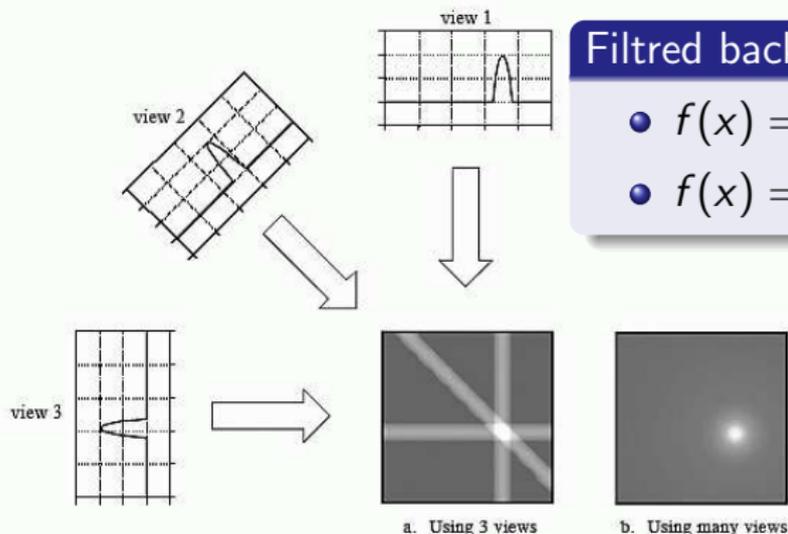


FIGURE 25-16

Backprojection. Backprojection reconstructs an image by taking each view and *smearing* it along the path it was originally acquired. The resulting image is a blurry version of the correct image.

Parallel geometry problems

Some problems with parallel geometry

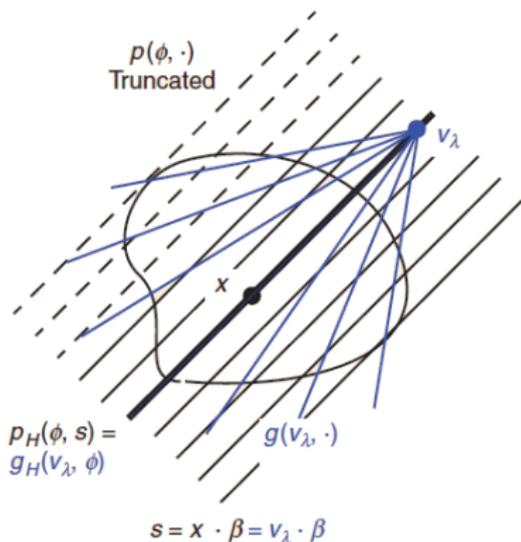
- The X-ray source doesn't cast parallel beams
- The parallel formulas require **all projections** $p(\phi, s)$
- This geometry is not adapted to data truncation

Work flow

- To adopt a new geometry called **fanbeam** geometry
- **Rebinning** truncated parallel projections into fanbeam
- To apply **Hilbert equality** to evaluate $p(\phi, s)$

Fanbeam geometry

Projection and Hilbert equality



Projection

$$g(\lambda, \phi) = \int_0^\infty f(v(\lambda) + l\alpha) dl$$

$$\alpha = (\cos \phi, \sin \phi)$$

$$\beta = (-\sin \phi, \cos \phi)$$

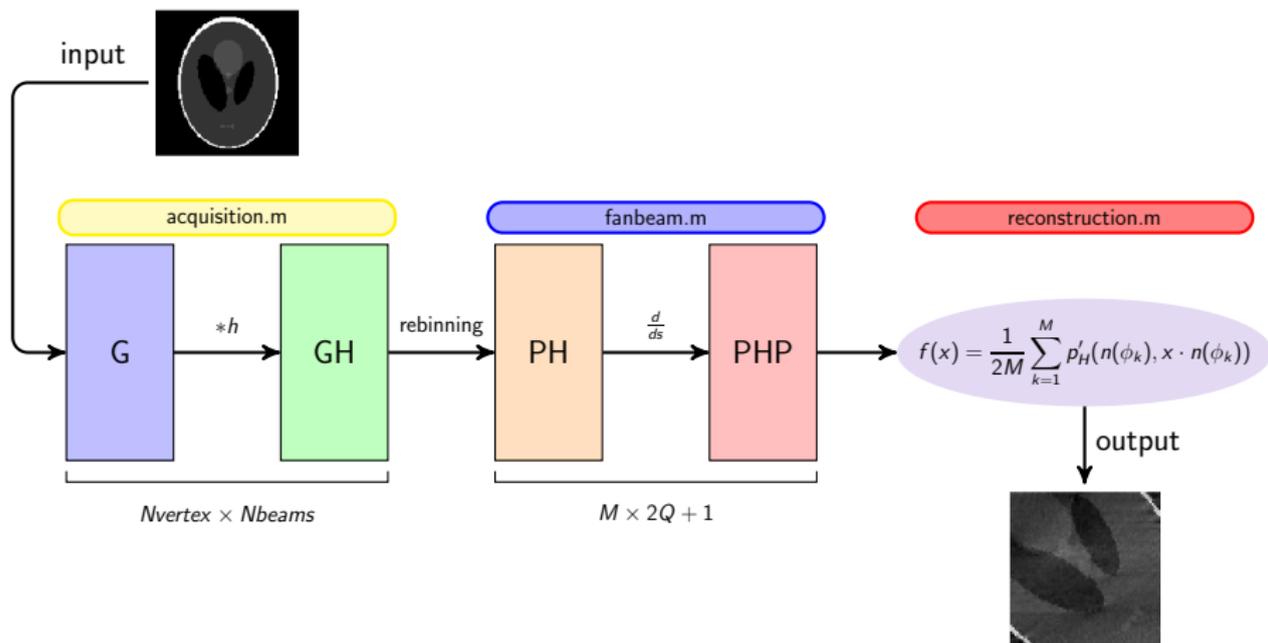
Hilbert equality

$$p_H(\phi, s) = g_H(v_\lambda, \phi), s = v_\lambda \cdot \beta$$

Contents

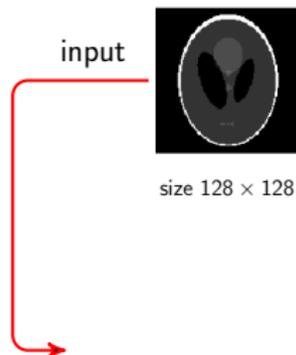
- 1 Introduction
- 2 Theory
- 3 Implementation**
 - Acquisition
 - Rebinning
 - Reconstruction
- 4 Results and analysis
- 5 Conclusion

Global vision of the architecture



Focus on the CT imaging process

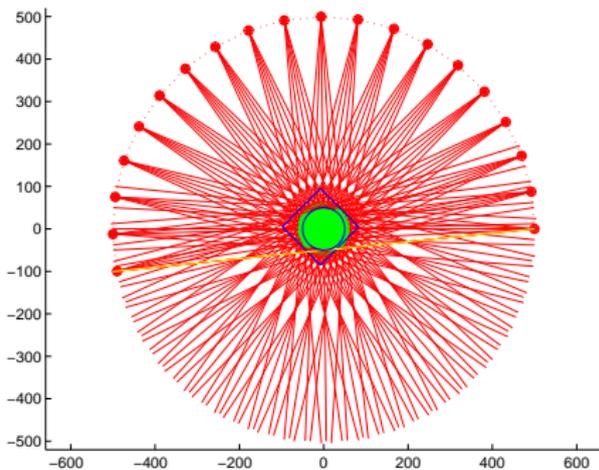
acquisition.m



Data available

Focus on the CT imaging process

acquisition.m

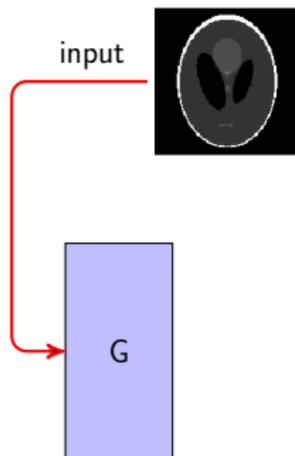


Data required

- image's position
- $R_{traj} = 150$
- $N_{vertex} = 512$
- $N_{beams} = 1024$
- $\gamma_m = \arcsin\left(\frac{FOV}{R_{traj}}\right) \simeq 20^\circ$
- $FOV = 50$
- integrale discretization=256

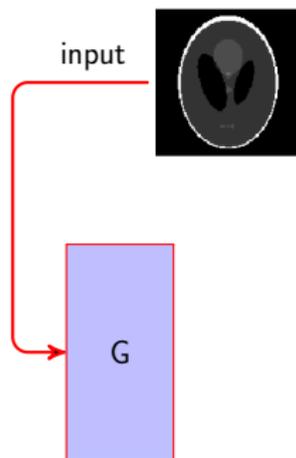
Focus on the CT imaging process

acquisition.m



Focus on the CT imaging process

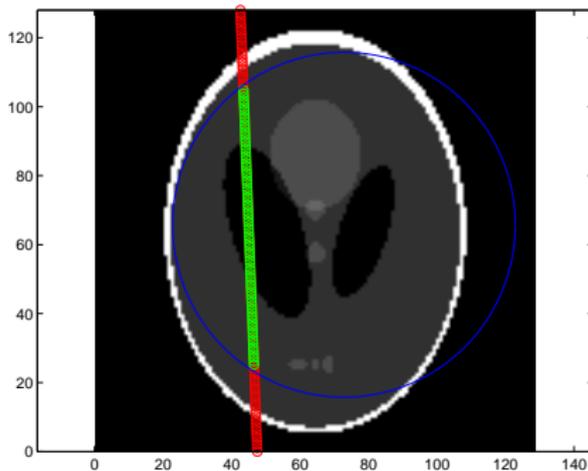
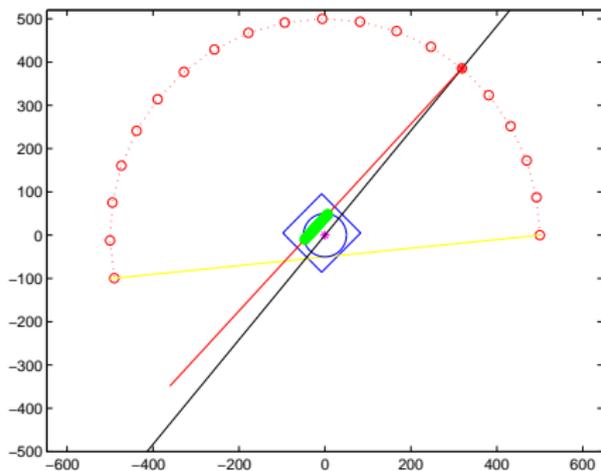
acquisition.m



Calculation of $g(\lambda_i, \alpha_j)$

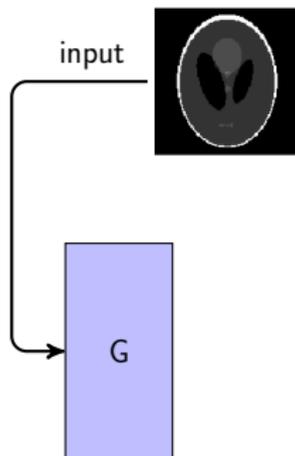
Focus on the CT imaging process

acquisition.m



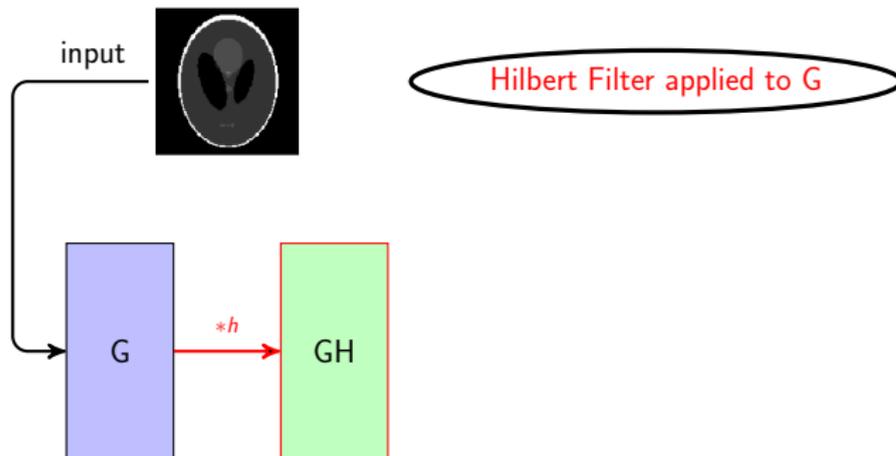
Focus on the CT imaging process

acquisition.m



Focus on the CT imaging process

acquisition.m

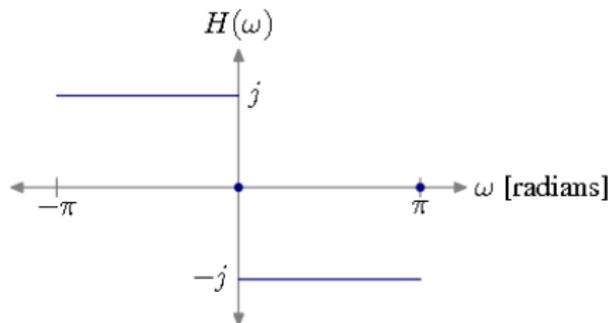


Filtered projections

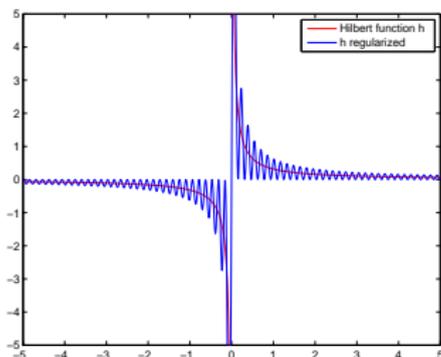
acquisition.m

Numerical problems

- Calculation of $g_H = g * h$ unstable around zero
- Solution \implies regularization

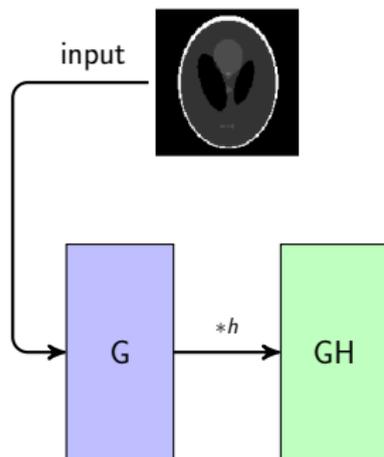


DTFT of the ideal discrete Hilbert transform filter for $-\pi < \omega \leq \pi$



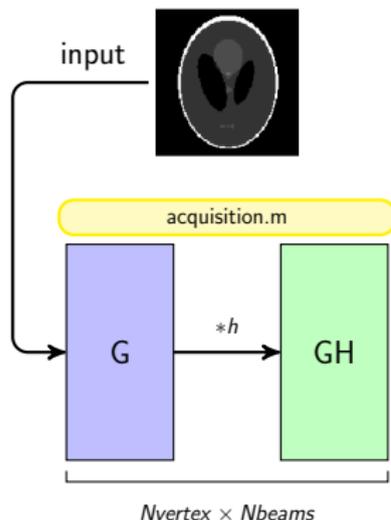
Focus on the CT imaging process

fanbeam.m



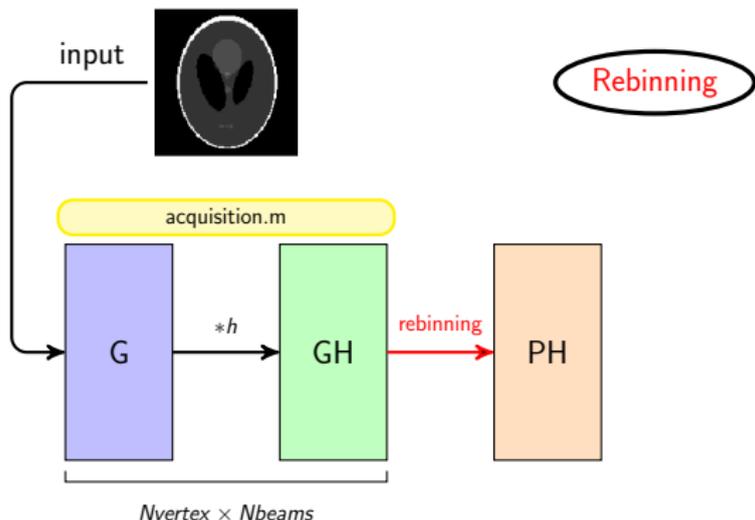
Focus on the CT imaging process

fanbeam.m



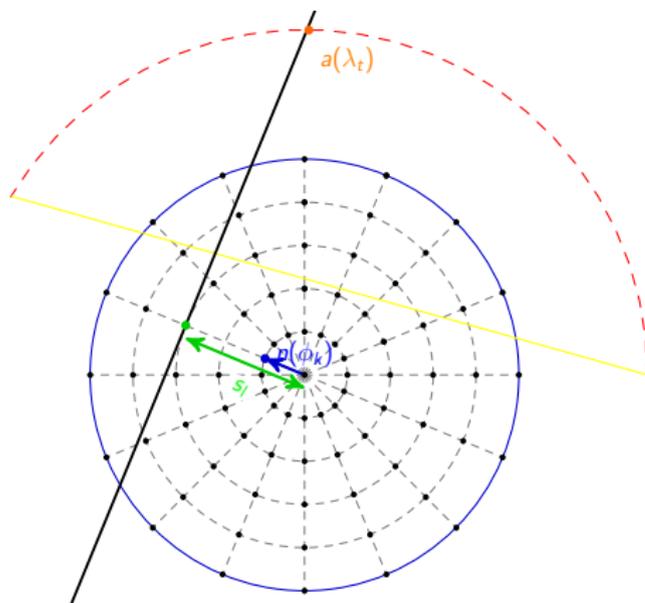
Rebinning : fanbeam geometry to parallel geometry

fanbeam.m



Rebinning : fanbeam geometry to parallel geometry

Discretized plan

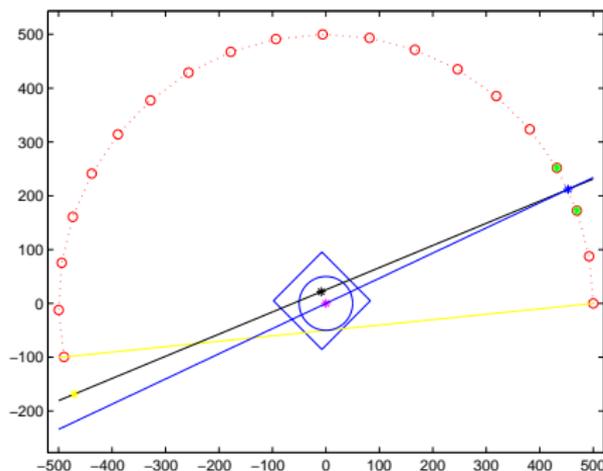


Discretization

- $\phi_k = k \frac{\pi}{M}, k = 0..M - 1$
- $s_l = l \frac{RROI}{Q}, l = -Q..Q$
- $Q = 512$
- $M = 1607$

Rebinning : fanbeam geometry to parallel geometry

The rebinning method step by step

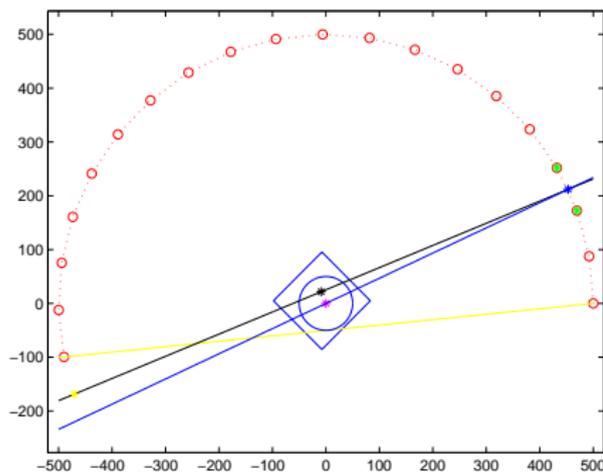


Steps

- 1 Intersection $a(\lambda_t)$ of $(n(\phi_k), s_l)$ with path
- 2 Determine the angle α_t between two lines
- 3 Give a bound of $\lambda_i \leq \lambda_t \leq \lambda_{i+1}$ and $\alpha_i \leq \alpha_t \leq \alpha_{i+1}$

Rebinning : fanbeam geometry to parallel geometry

The rebinning method step by step

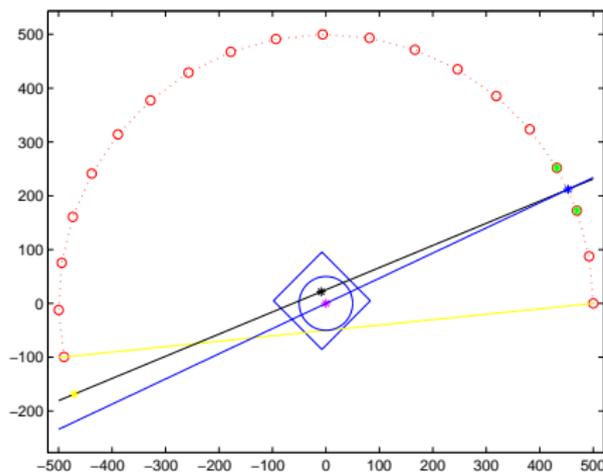


Steps

- 1 Intersection $a(\lambda_t)$ of $(n(\phi_k), s_l)$ with path
- 2 Determine the angle α_t between two lines
- 3 Give a bound of $\lambda_i \leq \lambda_t \leq \lambda_{i+1}$ and $\alpha_i \leq \alpha_t \leq \alpha_{i+1}$

Rebinning : fanbeam geometry to parallel geometry

The rebinning method step by step

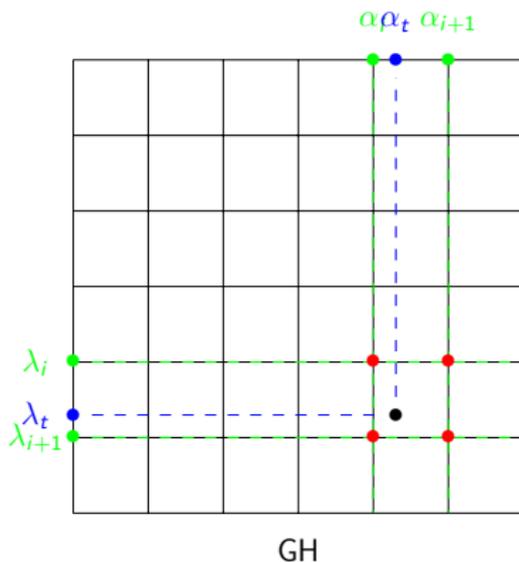
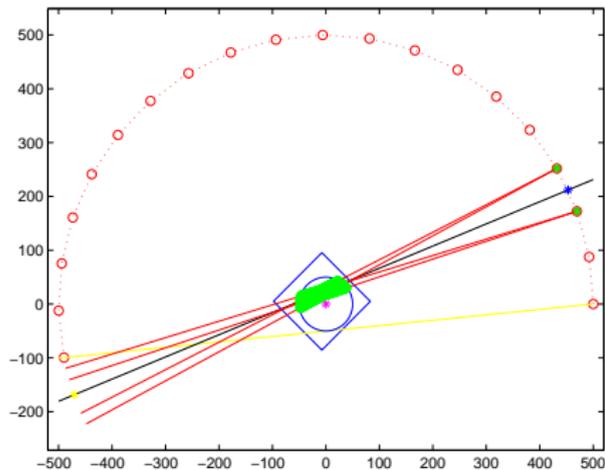


Steps

- 1 Intersection $a(\lambda_t)$ of $(n(\phi_k), s_l)$ with path
- 2 Determine the angle α_t between two lines
- 3 Give a bound of $\lambda_i \leq \lambda_t \leq \lambda_{i+1}$ and $\alpha_i \leq \alpha_t \leq \alpha_{i+1}$

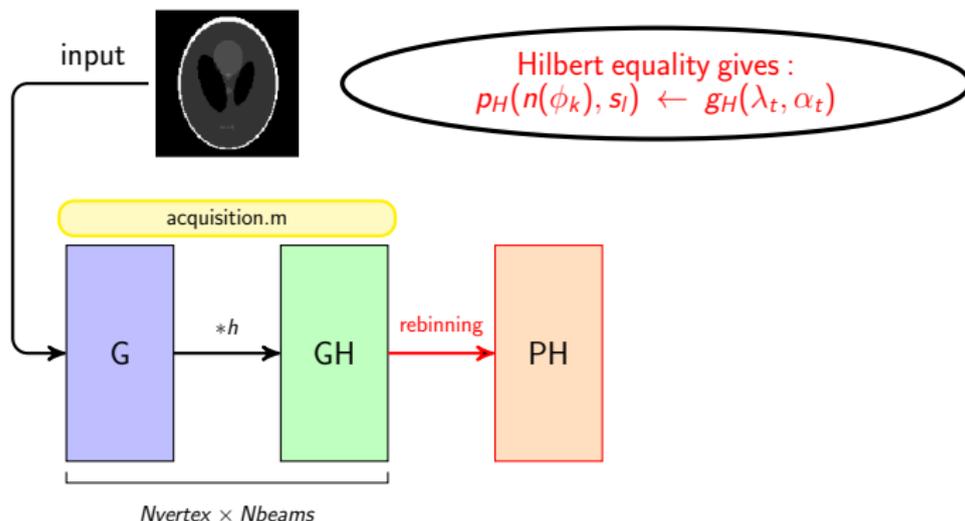
Rebinning : fanbeam geometry to parallel geometry

Bilinear interpolation of $g_H(\lambda_t, \alpha_t)$



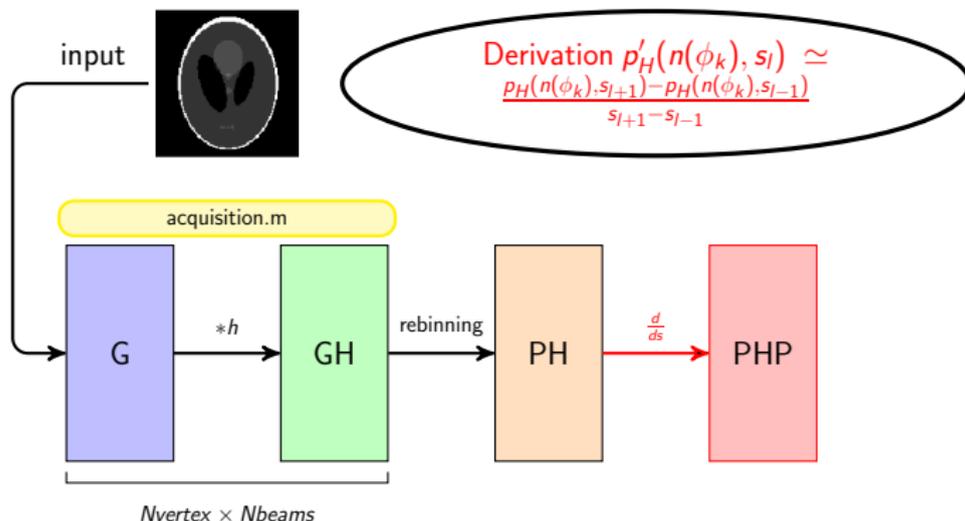
Rebinning : fanbeam geometry to parallel geometry

fanbeam.m



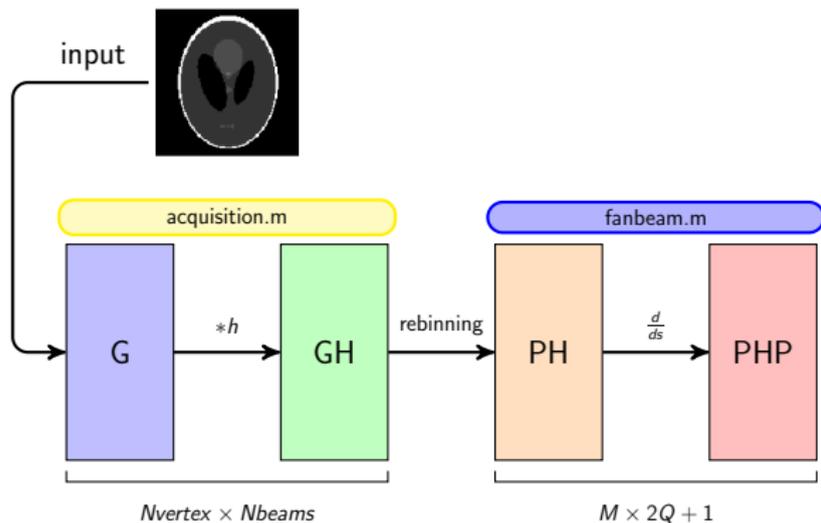
Rebinning : fanbeam geometry to parallel geometry

fanbeam.m



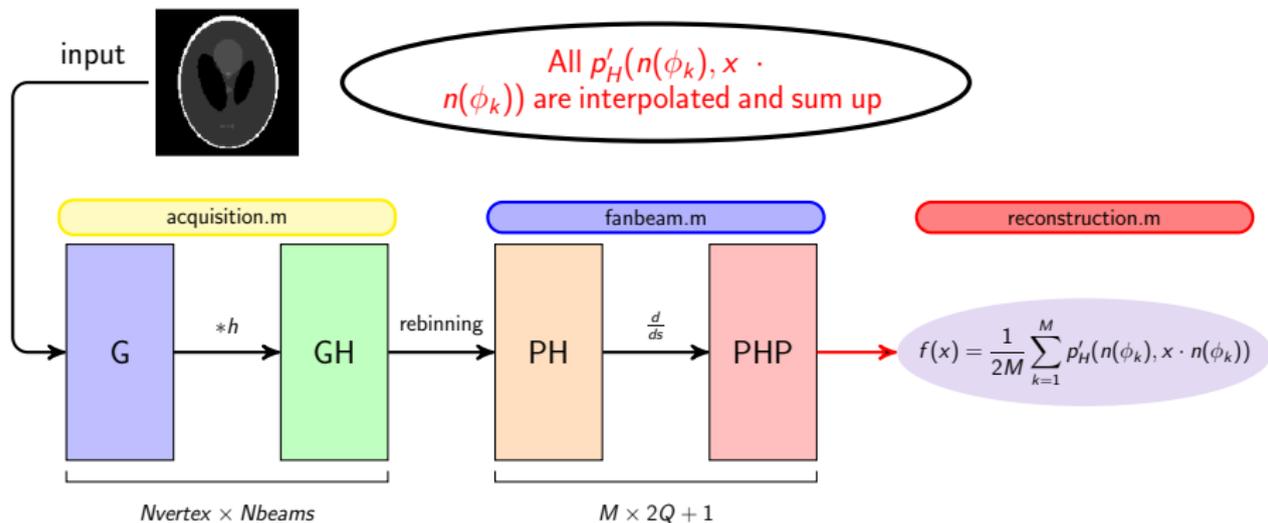
Rebinning : fanbeam geometry to parallel geometry

fanbeam.m



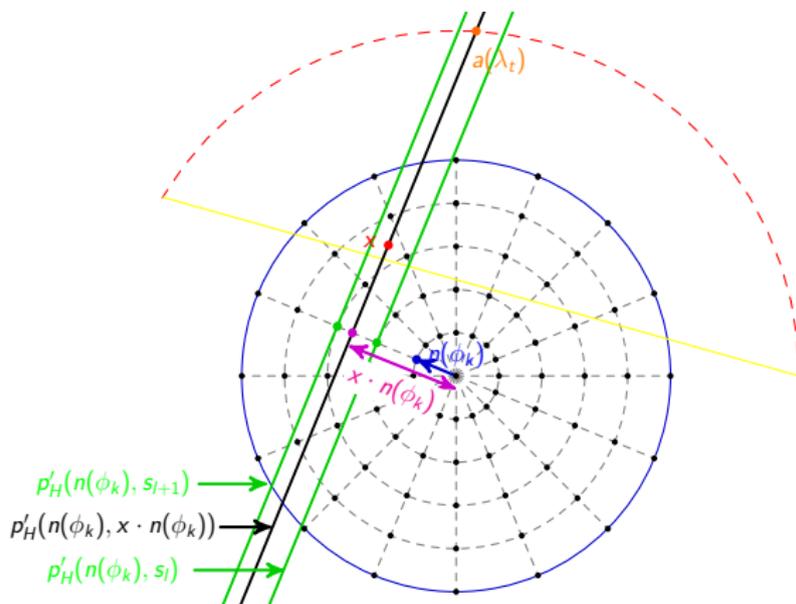
Reconstruction

reconstruction.m



Reconstruction

Interpolation



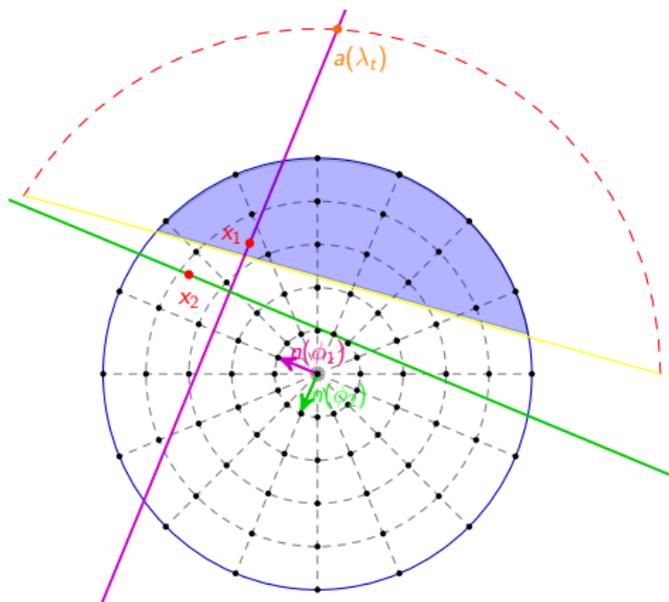
Linear interpolation

$$y = x \cdot n(\phi_k)$$

$$p'_H(n(\phi_k), y) \simeq \frac{(y-s_l)}{s_{l+1}-s_l} p'_H(n(\phi_k), s_{l+1}) + \frac{(s_{l+1}-y)}{s_{l+1}-s_l} p'_H(n(\phi_k), s_l)$$

Reconstruction

Condition of accurate reconstruction

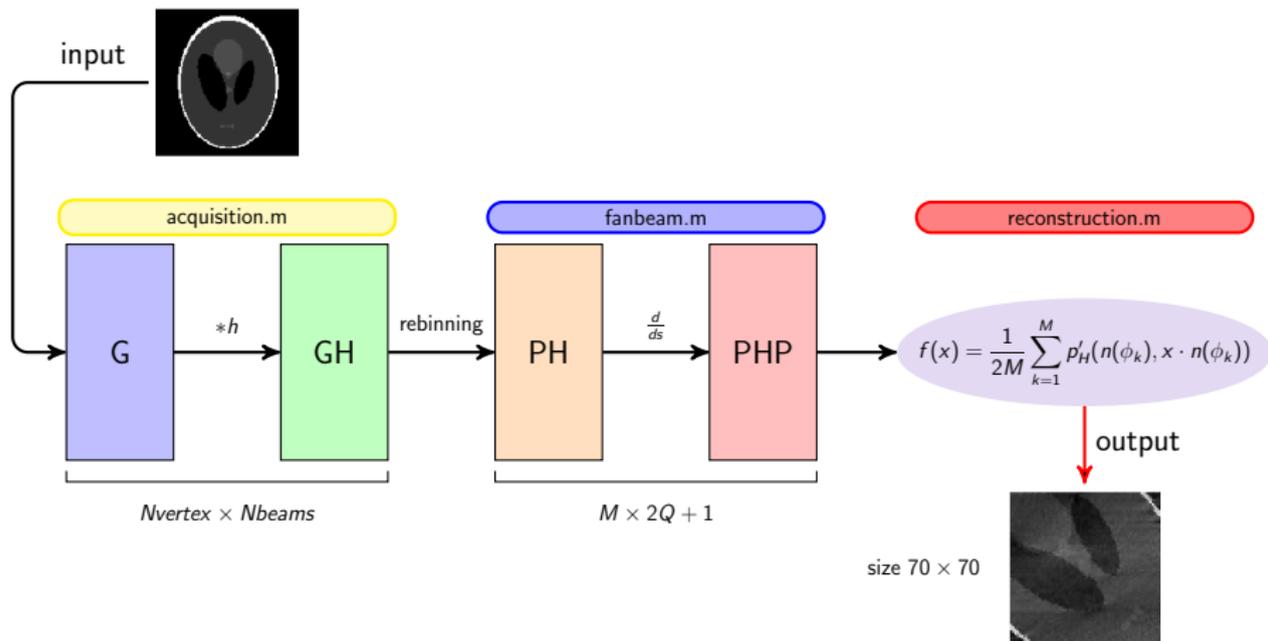


Fanbeam data condition

The point x can be reconstructed from complete fanbeam projections provided a fanbeam vertex can be found on each line passing through x

Reconstruction

reconstruction.m

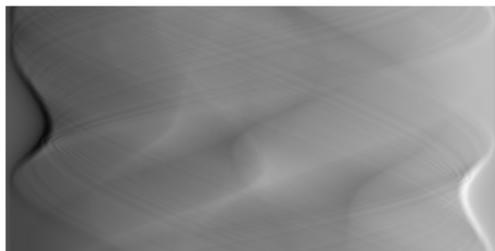
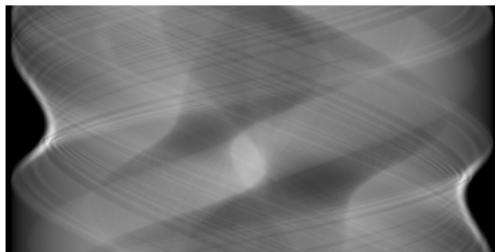


Contents

- 1 Introduction
- 2 Theory
- 3 Implementation
- 4 Results and analysis**
- 5 Conclusion

CT imaging results

acquisition.m : sinogram and filtered sinogram



Fanbeam measures

- On the top the sinogram
 $G = (g(\lambda_i, \alpha_j))_{i,j}$
- On the bottom the filtered sinogram after applying the Hilbert filter h ,
 $GH = (g_H(\lambda_i, n(\alpha_j)))_{i,j}$

Rebinning results

fanbeam.m : PH and PHP visualization



Parallel geometry

- On the left

$$PH = (p_H(n(\phi_k), s_l))_{k,l}$$

- On the right

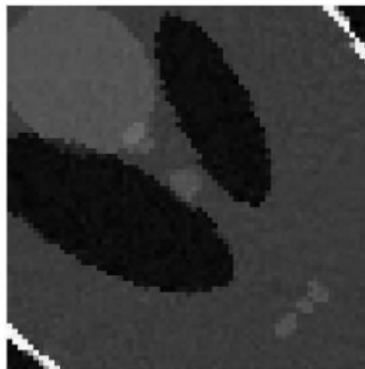
$$PHP = (p'_H(n(\phi_k), s_l))_{k,l}$$

Reconstruction results

reconstruction.m : reconstructed image display



Original image



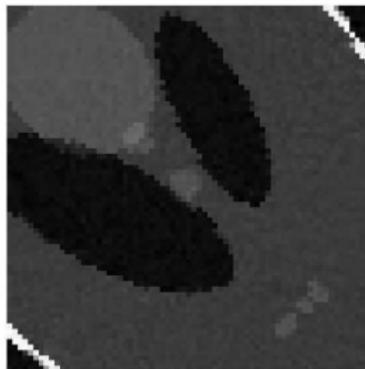
Reconstructed image

Reconstruction results

reconstruction.m : reconstructed image display



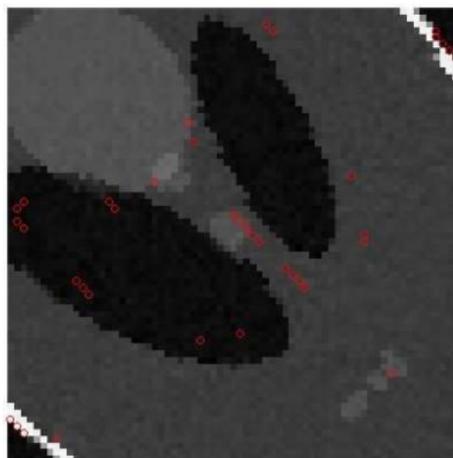
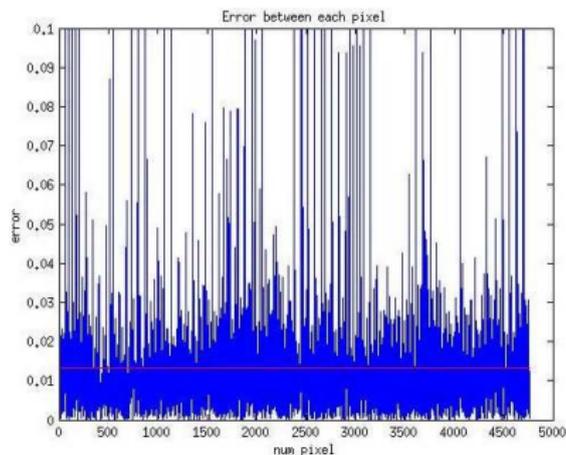
Original image



Reconstructed image

Reconstruction results

Error calculation



mean/pixel	standart deviation	quadratic	max	$> 10^{-1}$
1, $33 \cdot 10^{-2}$	2, $6 \cdot 10^{-2}$	2,01	0,62	38/4900

Contents

- 1 Introduction
- 2 Theory
- 3 Implementation
- 4 Results and analysis
- 5 Conclusion
 - Personal record
 - Future improvements

Conclusion

Personal record

Scientific point of view

- Discover of a new field of science applied to health
- Learn a new mathematical theory and its recent advances
- Fight against numerical problems
- Acceleration of the execution of programs

Human point of view

- Pluridisciplinary within research teams
- Partnership with companies
- Business dimensions

Conclusion

Future improvements

Improvements

- Report will be use to teaching aid
- C++ implementation, real time
- Adaptation of this algorithm in 3D reconstruction
- Extent to fanbeam truncated data \Rightarrow virtual fanbeam
- Integration in Surgivisio's softwares

Thank you for your attention

